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# Final Exam , MTH 221, Spring 2015

Ayman Badawi

**QUESTION 1. ( 10 points)**. Find the solution set for the the following system of linear equations

$$x_1 + 2x_2 = 1 - x_3$$
$$x_2 = 2 - x_3$$
$$x_1 + 3x_2 = 3 - 2x_3$$

**QUESTION 2.**

**(4 points)**. Given $S = span\{(1,0,0,1),(1,1,0,1),(1,1,1,1)\}$. Use Gram-Schmidt Algorithm to find an orthogonal basis for $S$.

**QUESTION 3. ( 10 points)** Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}.$$

(i) Find the inverse matrix of $A$ if it exists.

(ii) Find the inverse matrix of $A^T$ if it exists.

(iii) Find the third column of the inverse matrix of $AA^T$ if it exists.

**QUESTION 4. (6 points)**. Let $T : R^3 \longrightarrow R^3$ such that $T(a,b,c) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Then $T$ is a linear transformation (DO NOT SHOW THAT).

(i) Find dim(Range) and write the Range as a span of a basis.

(ii) Does the point $Badawi = (4,5,0)$ belong to the Range of $T$? If yes, find a point, say $Ayman = (a,b,c)$, such that $T(Ayman) = Badawi$

**QUESTION 5. (4 points)**. Imagine that $K$ is a subspace and $\{k_1, k_2\}$ is a basis for $K$. Is $\{k_1, k_1 + k_2\}$ a basis for $K$? Convince me (briefly) that your answer is acceptable.

**QUESTION 6. (9 points)** For each of the below, if the subset $S$ is a subspace, then rewrite it as a span of some basis, and tell me its dimension. If not a subspace, then give a counter-example.

(i) $S = \{(a, b) \in R^2 \mid (a, b) \text{ is orthogonal to } (2, -1)\}$

(ii) $S = \left\{(a, b) \in R^2 \mid a^2 + b^2 \leq 1\right\}$

(iii) $S = \{f(x) \in P_3 \mid f(0) = 3\}$

**QUESTION 7. (11 points)**.

$$A = \begin{bmatrix} a & b & c & d & e \\ f & g & h & i & j \\ k & l & m & n & o \\ p & q & r & s & t \\ u & v & w & x & y \end{bmatrix}$$

and suppose that det$(A) = \pi$.

(i) Find det$(A^{-1})$, $det(2A^T)$,

(ii) Find the determinant of $B = \begin{bmatrix} 1 & b & c & d & e \\ 0 & 1 & h & i & j \\ 0 & 0 & 1 & n & o \\ 0 & 0 & 0 & 1 & t \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$,

For the matrix $B$ above, what are the eigenvalues of $B$? Assume that $B$ is is diagnolizable (Do not show that), for each eigenvalue $a$ of B find $.dim(E_a)$

(iii) Find the determinant of $\begin{bmatrix} 1 & b & c & d & e \\ 0 & 1 & h & i & j \\ 0 & 0 & 1 & n & o \\ 0 & 0 & 0 & 1 & t \\ 1 & b & c & d & 3+e \end{bmatrix}$.

**QUESTION 8. (16 points) Determine whether each statement is true or false and give a brief justification for your answer (should not exceed two <u>CLEAR MEANINGFUL</u> lines)**

(i) If $A$ is a $3 \times 3$ invertible matrix, then $A$ is diagnolizable.

(ii) It is impossible to construct a linear transformation $T : R^2 \rightarrow R^4$ such that $dim(Range(T)) = 3$.

(iii) If $A$ is a $4 \times 4$ matrix and 1 is an eigenvalue of $A$, then there is a nonzero $4 \times 10$ matrix $B$, such that $AB = B$.

(iv) If $T : R^3 \to R$ is a linear transformation and $T(1, 4, 7) = \pi$, then $dim(Ker(T)) = 2$.

(v) If $A$ is a $4 \times 5$ matrix and $Rank(A) = 4$, then the system $AX = \begin{bmatrix} 2 \\ 5 \\ 7 \\ 9 \end{bmatrix}$ has infinitely many solutions

(vi) If $A$ is a $3 \times 3$ matrix such that $det(A) = 0$, then the system $AX = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$ has infinitely many solutions

(vii) If $A$ is a $4 \times 4$ matrix and the system $AX = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 7 \end{bmatrix}$ is inconsistent (i.e., it has no solution), then the system $AX = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ has infinitely many solutions.

(viii) If $A$ is a $4 \times 4$ matrix and $C_A(x) = x^2(x - 3)^2$, then the system $AX = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ has infinitely many solutions.

**Faculty information**

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